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A DIFFUSION MODEL FOR DROPLET DEPOSITION IN GAS/LIQUID ANNULAR FLOW

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Abstract—Droplet deposition in vertical gas/liquid annular flow is modeled by considering dispersion from a ring source on the wall. Flow situations such as fully developed flow or droplet concentrations downstream of a film removal unit are then described by appropriate distributions of ring sources at the wall.

Key Words: annular flow, gas/liquid flow, droplet deposition, two-phase flow, droplet concentration profiles.

1. INTRODUCTION

For annular gas/liquid flows in a vertical pipe, part of the liquid moves as a film on the wall and part is entrained as droplets in the high-velocity gas core (Hewitt & Hall-Taylor 1970). Droplets are formed by the atomization of small wavelets riding on the top of large-amplitude flow surges in the wall film and are redeposited back on the film downstream. A critical problem in analyzing annular flows is understanding the rate process which governs the deposition of entrained droplets.

The ability of droplets to respond to gas phase turbulence is represented by the reciprocal time constant, β defined as

$$\beta = \frac{3C_D\rho_G}{4D_p\rho_p}|\mathbf{u}_r|. \qquad [1]$$

For the Stokes flow regime,

$$\beta = \frac{18\mu_G}{D_p^2\rho_p}; \qquad [2]$$

where C_d is the drag coefficient on the drop, ρ_G and ρ_p are the density of the gas and the drop, D_p is the diameter of the drop, $\mathbf{u}_r$ is the relative velocity between the drop and the surrounding gas and μ_G is the viscosity of the gas. For annular flows, $\tau^+ = u_*^2/\nu_G\beta$ is $\gg 20$ (where ν_G is the kinematic viscosity of the gas). As indicated in a review article by McCoy & Hanratty (1975), such particles have stopping distances larger than the thickness of the viscous wall-region. Consequently, the non-homogeneity of the turbulence near the wall can be ignored. The droplets take on turbulence characteristics by reacting to the approximately homogeneous fluid velocity fluctuations in the gas outside the viscous wall layer. The square of the radial turbulent velocity fluctuations, v_p^2, and the particle diffusivity can be defined as $\epsilon_p = \overline{v_p^2}\tau_p$, where τ_p is the Lagrangian time scale of the particle. The dispersion of particles in the gas phase might be described by a diffusion model if the length scale characterizing particle motion, $(\overline{v_p^2})^{1/2}\tau_p$, is less than the pipe radius.

Lee *et al.* (1989a) explored the possibility of using a homogeneous diffusion model by considering the highly idealized situation of the dispersion and deposition of droplets of uniform size that were injected from a small orifice at the center of the pipe at the same velocity as the gas. Deposition results were compared with a solution of the diffusion equation. Non-homogeneities were ignored so that ϵ_p, $\overline{v_p^2}$ and the streamwise velocity were assumed independent of radial position.

Of particular importance was the formulation of the boundary condition at the wall, which was considered a perfect absorber. For a process involving only molecular diffusion the assumption of perfect absorption implies a zero concentration at the wall. However, this is not the case for droplet deposition for which the length scale of the droplet motion can be large compared to the scale

characterizing the variation of the droplet concentration. Thus, the droplet concentration at the wall has a finite value, given by

$$-\epsilon_p \frac{\partial C}{\partial r}\bigg|_{r=a} = f\sqrt{\frac{2}{\pi}}(\overline{v_p^2})^{1/2} C\bigg|_{r=a}. \quad [3]$$

Here, f is the fraction of the droplets that are moving toward the wall and C is the concentration of the droplets (number per unit volume). In terms of this formulation the mass transfer resistance can be considered as the sum of two processes in series: diffusion to the wall and free flight deposition on the wall.

This paper extends the diffusion model of Lee *et al.* (1989a) to the case of annular gas/liquid flow. In particular, droplets are pictured to enter the flow field from a differential ring source on the pipe wall. All of the droplets are assumed to redeposit on the wall (without bouncing) downstream of the ring source. The diffusion equation is solved to describe the behavior of one of these wall sources. The droplet field for a particular annular flow situation is then represented as resulting from the contributions of a number of such sources.

The deposition rate per unit area, R_D, is usually correlated with droplet concentration, C_B, by the following equation:

$$R_D = k_D C_B, \quad [4]$$

where k_D is a deposition constant. A number of researchers have reported measurements of k_D. The works of Cousins & Hewitt (1968), Andreussi (1983) and Schadel *et al.* (1990) are of particular interest in this work. The approach used in this paper is more closely related to that used by James *et al.* (1980) and Andreussi & Azzopardi (1983), who described the droplet transfer process in terms of a function describing the lifetime distribution of droplets originating from a ring source on the wall.

A number of results emerge from the analysis which appear to be useful in understanding measurements of k_D. For fully developed annular flow a relatively flat concentration with a slight maximum close to the wall is calculated. Thus, the free flight to the wall controls deposition and, according to [3], k_D should scale roughly with $(\overline{v_p^2})^{1/2}$. For the situation considered by Cousins & Hewitt (1968), where deposition is measured downstream of a film withdrawal unit, the concentration profile changes from a flat to a diffuse concentration profile. Thus, the diffusion resistance becomes increasingly important downstream of the withdrawal unit and k_D is predicted to decrease.

2. DESCRIPTION OF TURBULENT DIFFUSION

Taylor (1921) described the turbulent diffusion of fluid particles, originating from a point source in homogeneous turbulence, in terms of a turbulent intensity $\overline{v^2}$ and a time scale:

$$\tau = \int_0^\infty \frac{\overline{v(t)v(t+\tau)}}{\overline{v^2(t)}}\,d\tau. \quad [5]$$

Taylor's theory can also be used to describe the turbulent diffusion of a particulate. However, because the particle has larger inertia than the fluid it will have a smaller turbulent intensity and a larger time scale. Theoretical predictions of particle intensities and time scales have been given by Reeks (1977) and Pismen & Nir (1978). Experimental measurements of these quantities have been made by Vames & Hanratty (1988) and Lee *et al.* (1989b) for droplet sizes typical of annular flow.

Using Taylor's analysis for a homogeneous isotropic field, the variation of the mean square displacement of the particles with time is given as

$$\frac{d\overline{X_p^2}}{dt} = 2\overline{v_p^2}\int_0^t R(t')\,dt', \quad [6]$$

where

$$R(t') = \frac{\overline{v_p(0)v_p(t')}}{\overline{v_p^2(0)}}. \quad [7]$$

The Lagrangian autocorrelation can be approximated by

$$R(t') = \exp\left(-\frac{t'}{\tau_p}\right), \quad [8]$$

where $\overline{v_p^2}$ and τ_p are the turbulent intensity and time scale of the particles, respectively.

Batchelor (1949) showed that if the distribution function describing $\overline{X_p^2}$ is Gaussian, then the concentration of the turbulent diffusing particles from an infinitesimal source is given by a solution to the diffusion equation:

$$\frac{\partial C}{\partial t} = \epsilon_p \nabla^2 C, \quad [9]$$

where

$$\epsilon_p(t) = \frac{1}{2}\frac{d\overline{X_p^2}}{dt} \quad [10]$$

In this paper the infinitesimal source will be considered to be a differential ring on a wall. The approach used is similar to that pursued by Hanratty (1956, 1958), Hanratty & Flint (1956) and Johnk & Hanratty (1962) to describe molecular transport of heat and momentum from a wall source.

3. APPLICATION TO VERTICAL ANNULAR FLOW

An idealized picture of vertical annular flow is shown in figure 1. The droplets enter the gas phase from a differential ring source located at the top of the film. The film height is assumed to be

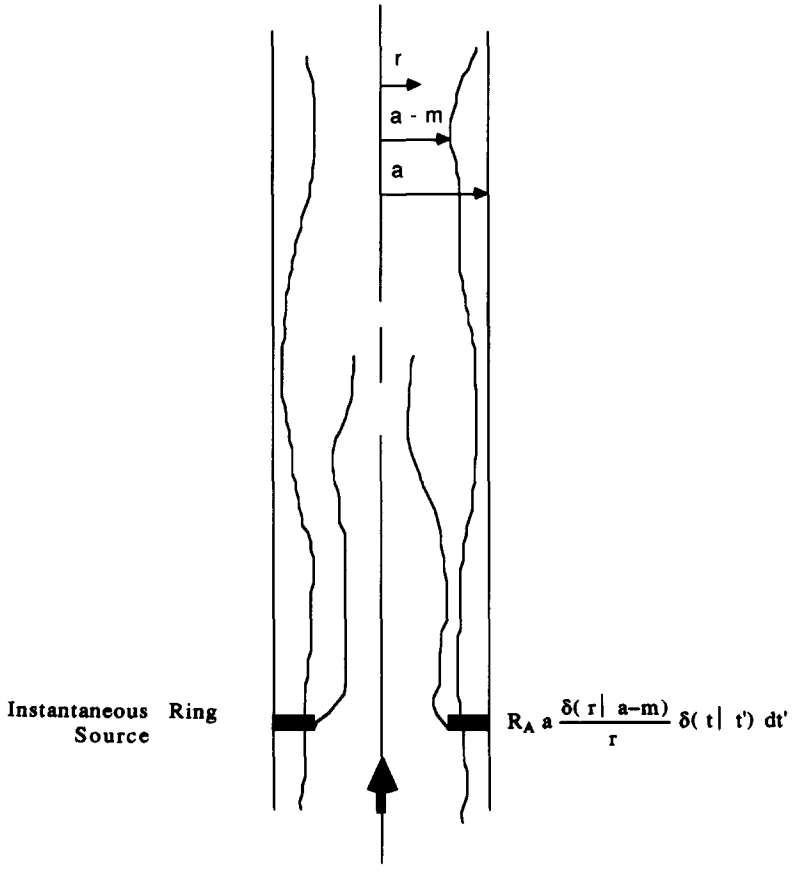

Figure 1. Idealized picture of annular flow showing the location of the differential ring source.

constant and very thin ($m \ll a$), so the droplets can be assumed to originate from the pipe wall. The droplets are entrained into the gas phase and eventually deposited back on the film or pipe wall at some location downstream. This paper describes the behavior of one instantaneous differential ring source by solving [9]. The fully developed condition is calculated by considering the contribution of many ring sources.

In annular flow the atomization process produces a distribution of droplet sizes and ejection velocities. Although several researchers have produced experimental data on the drop size distribution in annular flow, a reliable correlation is not available. Therefore, it will be assumed that the droplet field with a velocity equal to their turbulent velocity intensity $(\overline{v_p^2})^{1/2}$. In this sense, they are assumed initially to be fully entrained in the fluid turbulence.

Lee *et al.* (1989a) pointed out that special attention must be paid to the boundary condition used to solving [9]. For particles with large stopping distances the radiation boundary condition [3] is appropriate. The particle diffusivity, ϵ_p, is given by [5]–[8] and [10] as

$$\epsilon_p(t) = \overline{v_p^2}\tau_p\left[1 - \exp\left(-\frac{t}{\tau_p}\right)\right]. \quad [11]$$

The fraction of particles moving towards the wall, f, is assumed to take the form

$$f(t) = \frac{1}{2}\left[1 - \exp\left(-\frac{t}{\tau_p}\right)\right]. \quad [12]$$

Initially all of the particles are moving away from the wall so $f(0) = 0$. The function $f(t)$ approaches 1/2 asymptotically at large times with the same time dependence as the diffusivity. The choice of this function is made for mathematical convenience, although the choice is physically reasonable since it would not be expected that $f(t)$ equals 1/2 until the dispersing droplets have completely obtained the haphazard motion characteristic of diffusion at large times.

Two further assumptions are made in applying the diffusion model to annular flows. Droplet loadings are dilute enough so that droplet–droplet interactions can be ignored and that there is no effect of droplets on the turbulence of the gas phase. The variations in the gas mean velocity profile is ignored; i.e. a plug flow is assumed.

The approach taken here includes concepts not previously considered in diffusion models for droplet deposition. The work of Hutchinson *et al.* (1971) is the most closely related. Hutchinson *et al.* (1971) calculated the fraction of particles approaching the wall region as a solution to the time independent diffusion equation for a disk source and assumed a zero concentration near the wall. The fraction of particles that penetrate the wall region to deposit was found to be one for particle sizes and flow conditions typical of annular flow. The present model differs from that of Hutchinson *et al.* by treating the time dependent diffusion of particles from instantaneous ring sources on the wall. Additional differences are that the boundary condition [3] allows for a non-zero concentration near the wall, as is found experimentally in annular flow, and that the diffusion coefficient of the droplets and the fluid are taken to be equal in order to be consistent with recent studies cited in the next section.

4. SCALING

Equation [9] and the boundary condition [3] are made dimensionless by using the following scaling:

$$r \to \frac{r}{a}, \quad t \to \frac{tu_*}{a}, \quad \epsilon_p \to \frac{\epsilon_p}{u_* a}, \quad (\overline{v_p^2})^{1/2} \to \frac{(\overline{v_p^2})^{1/2}}{u_*}, \quad C \to \frac{Cu_*}{R_A}, \quad [13]$$

where R_A is the flux of droplets entering the pipe at the wall; u_* is the friction velocity of the gas phase, defined as the square root of the ratio of the interfacial stress and gas density as follows:

$$u_* = \sqrt{\frac{\tau_i}{\rho_G}}. \quad [14]$$

Vames & Hanratty (1988) reviewed turbulence measurements in a pipe to find that the fluid diffusivity, ϵ_f, integral time scale, τ_{Lf}, and the turbulent intensity correlate with the friction velocity as follows:

$$\frac{\epsilon_f}{u_* 2a} = 0.037, \quad [15]$$

$$\frac{\tau_{Lf} u_*}{2a} = 0.046 \quad [16]$$

and

$$(\overline{u^2})^{1/2} \cong 0.9 u_* \quad \text{at } r \cong a. \quad [17]$$

The scaling [13] indicates that the dimensionless particle diffusivity, time scale and turbulent intensity are proportional to the ratios of the values for the particle and the fluid.

For situations in which the relative velocity between the particle and fluid is small, $(U_G - U_p)/(\overline{u^2})^{1/2} < 0.5$, the theory of Reeks (1977) and measurements (Vames & Hanratty 1988; Lee *et al.* 1989a; Young 1989) indicate that the particle diffusivity, ϵ_p, is approximately equal to the fluid diffusivity, ϵ_f, and the mean square turbulent fluctuations can be approximated by

$$\frac{(v_p^2)^{1/2}}{(u^2)^{1/2}} = \left(\frac{\beta \tau_{Lf}}{0.7 + \beta \tau_{Lf}} \right)^{1/2}. \quad [18]$$

The ratio of τ_p/τ_{Lf} can be obtained from [18] by noting that $\epsilon_p/\epsilon_f = \overline{v_p^2} \tau_p / \overline{u^2} \tau_{Lf}$.

For the case where particle relative velocity is small, one dimensionless parameter, $\beta \tau_{Lf}$, enters the problem. This parameter compares the inertial relaxation time of the particle, $1/\beta$, to the time scale of the fluid. As $\beta \tau_{Lf}$ decreases the ability of the particle to react to the fluctuations in the fluid turbulence decreases. It is of interest to determine the range of $\beta \tau_{Lf}$ where the diffusion model being considered for droplet deposition is applicable to annular flow.

Andreussi & Azzopardi (1983) considered two mechanisms for droplet deposition in annular flow. The first is a diffusion mechanism, such as the one considered here, and the second a direct impaction mechanism, identified by Russell & Rogers (1972) and James *et al.* (1980). The direct impaction possibility arises when droplets with large inertia are ejected into the gas phase and do not change direction before they redeposit. These droplets will move in a straight line trajectory and deposit in a quadrant of the pipe opposite of the quadrant of ejection. Andreussi & Azzopardi (1983) presented the following dimensionless group as a criteria for determining the dominating mechanism for deposition in annular flow:

$$G = \frac{\rho_L V D_p}{C_D \rho_G u_* D_t}, \quad [19]$$

where D_p is the mean drop diameter, D_t is the pipe diameter and V is the ejection velocity of the droplet from the film. The group G represents the ratio of the initial momentum to the fluid drag. The ejection velocity, V, was determined from the cine films of James *et al.* (1980) to scale with the friction velocity of the gas. The parameter $\beta \tau_{Lf}$ can be related to G. By assuming a Stokes drag law, the criterion of Andreussi & Azzopardi (1983) suggests that for values of $\beta \tau_{Lf} > 0.01$ a diffusion mechanism is operative. From [15], [17] and [18] this corresponds to a value of $(\overline{v_p^2})^{1/2} \tau_p / a = 0.70$, for which the turbulent length scale of the particles is roughly equal to the pipe radius.

5. BEHAVIOR OF ONE INSTANTANEOUS RING SOURCE

The behavior of one instantaneous ring source is described as the solution of the following non-dimensional diffusion equation:

$$\frac{\partial C(r,t)}{\partial t} = \frac{\epsilon_p (t - t')}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C(r,t)}{\partial r} \right) + \frac{\delta(r \mid 1)}{r} \delta(t \mid t') \, dt', \quad [20]$$

with boundary condition [3] applying. Time t' is the time the unit instantaneous source dt' is turned on at $r = 1$. The solution of [20] is obtained by standard means and is given as

$$C(r, t-t')\Big|_{\text{source}}^{\text{single}} = \sum_{n=1}^{\infty} \frac{J_0(\beta_n r)J_0(\beta_n)}{N_n} \exp[-\beta_n^2 K(t-t')]\,dt', \quad [21]$$

where

$$K(t-t') = \int_{t'}^{t} \frac{1}{2}\frac{d\overline{X_p^2}}{dt}\,dt. \quad [22]$$

The eigenvalues, β_n, are given by the transcendental equation

$$\beta_n J_1(\beta_n) - \sqrt{\frac{1}{2\pi}}\frac{(\overline{v^2})^{1/2}}{\epsilon_{p\infty}} J_0(\beta_n) = 0, \quad [23]$$

where $\epsilon_{p\infty}$ is the value of ϵ_p at large time.

For the dispersion of particles with $(U_G - U_p)/(\overline{u^2})^{1/2} < 0.5$, the particle diffusivity can be taken as the fluid diffusivity. Thus, only the inertial parameter, $\beta\tau_{Lf}$, enters the problem. Figure 2 shows calculated concentration profiles at various values of the dimensionless time. The value of the inertial parameter in this case equals one. The concentration has a finite value at the wall at all times. At times close to zero $f = 0$, so that there is no deposition at the wall. A maximum occurs at the wall and there is a diffusion of droplets away from the wall. As f takes on values different from zero, deposition occurs and there is diffusion both to the wall and away from the wall region. As a consequence, a peak in the droplet concentration profiles occurs close to the wall. As time increases the concentration profile becomes more diffuse. Eventually the maximum is at the pipe center and there is only diffusion toward the wall.

Figure 3 shows the unit source calculation for a value of $\beta\tau_{Lf} = 0.01$. The profiles are calculated at the same values of dimensionless time as in figure 2. The same qualitative behavior is found in figures 2 and 3. However, the time development is much slower, as would be expected for a particle with a larger time scale or, from [18], a smaller $(\overline{v_p^2})^{1/2}$.

It is of interest to calculate the fraction of the droplets deposited from the unit source in order to compare with the simple bulk deposition model [4]. The fraction of droplets originating from an instantaneous source that deposit is given as

$$F(t)\Big|_{\text{source}}^{\text{single}} = \frac{\int_{t'}^{t} -\epsilon_p(t-t')\dfrac{\partial C(r, t-t')}{\partial r}\,dt}{\int_{t'}^{\infty} -\epsilon_p(t-t')\dfrac{\partial C(r, t-t')}{\partial r}\,dt}. \quad [24]$$

Time can be converted to the distance travelled downstream if it is assumed that the droplets travel axially at a constant velocity. For the case of small relative velocity the particle axial velocity can be taken as the bulk gas velocity. In contrast, the calculated fraction depositing for a simple concentration driven rate process is given by the expression

$$F_D(z) = 1 - \exp\left(-\frac{2k_D}{U_G}\frac{z}{a}\right), \quad [25]$$

where k_D is the deposition constant defined in [4].

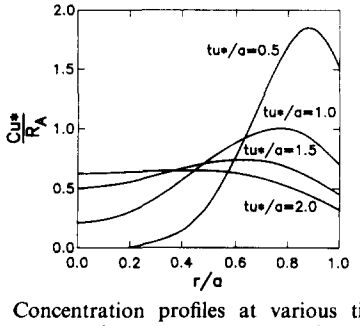

Figure 2. Concentration profiles at various times after turning on one instantaneous source ($\beta\tau_{Lf} = 1.0$).

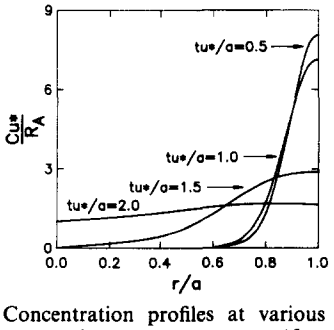

Figure 3. Concentration profiles at various times after turning on one instantaneous source ($\beta\tau_{Lf} = 0.010$).

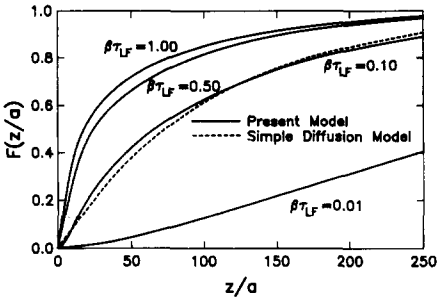

Figure 4. Effect of the inertial parameter, $\beta\tau_{Lf}$, on the fraction deposited and comparison with a simple diffusion model.

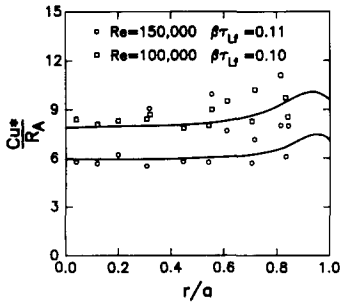

Figure 5. Comparison of the droplet concentration data of Gill *et al.* (1965) with the prediction of the present model.

Figure 4 compares the fraction depositing that is predicted by [24] and [25] at a gas Reynolds number (Re = 100,000) typical for annular flow experiments. For particles with smaller $\beta\tau_{Lf}$, i.e. particles with greater inertia, the time of residence in the gas phase increases. A value of $k_D = 0.15$ m/s, suggested by James *et al.* (1980), was used in calculations with [25]. In order to use the results from [24] it is necessary to evaluate $\beta\tau_{Lf}$. The Lagrangian time constant of the fluid is defined by [16] and the reciprocal time constant of the particle is obtained from [2]. In order to use [2] and [16], the droplet diameter and the friction velocity must be known. The mean drop size was calculated from a correlation presented by Azzopardi (1985). The friction velocity was obtained from a correlation presented by Asali *et al.* (1985). For the Re value considered in figure 4 a value of 0.1 is estimated for $\beta\tau_{Lf}$. The calculations for $\beta\tau_{Lf} = 0.1$ and the simple deposition model agree in magnitude. However, the two models give different functional forms for the droplet lifetime distribution functions.

6. RESULTS

A minimal amount of data is available on concentration profiles in annular flow. The most complete study was performed by Gill *et al.* (1965) by using an impact tube method. The droplet diffusion model that has been described is useful in interpreting these measured concentration profiles.

The contribution of many instantaneous sources is obtained by integration as follows:

$$C(r, t) = \sum_{n=1}^{\infty} \frac{J_0(\beta_n r) J_0(\beta_n)}{N_n} \int_0^t \exp[-\beta_n^2 K(t - t')]\, dt'. \quad [26]$$

The fully developed condition is obtained by taking the limit $t \to \infty$.

Gill *et al.* (1965) presented data at two different air flow rates for a range of liquid flows. Parameter $\beta\tau_{Lf}$ is estimated in the manner outlined in the previous section. The assumption of a dilute droplet loading was satisfied by selecting conditions where the correlation of Azzopardi (1985) indicated that the liquid flow rates has little effect on drop size.

Figure 5 presents a comparison of the concentration data and the prediction of the model. A relatively flat profile is calculated with a slight maximum near the wall. A simple model of the type used for molecular diffusion would use a diffusivity that is independent of time and a concentration of zero at the wall; a flat profile would not be predicted. Consideration of the time dependency of the turbulent diffusion process and the use of boundary condition [3] yields the correct interpretation of concentration profiles in vertical annular flow.

It is of interest to use this diffusion model to interpret reported measurements of the deposition constant, k_D, in annular flow. Lee *et al.* (1989a) considered the mass transfer resistance as the sum of two processes in series. A diffusive mass transfer coefficient, k_l, is defined by the following:

$$R_D = k_l[C_B - C(r/a = 1)], \quad [27]$$

where R_D is the flux of droplets depositing on the wall, C_B is the bulk concentration and $C(r/a = 1)$ is concentration near the wall. A free flight mass transfer coefficient is defined as

$$R_D = k_2 C(r = a). \quad [28]$$

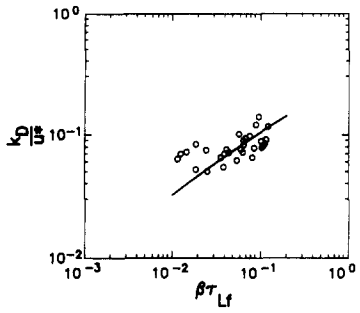

Figure 6. Comparison of the deposition constants measured by Schadel *et al.* (1990) for fully developed annular flow with the prediction of the present model.

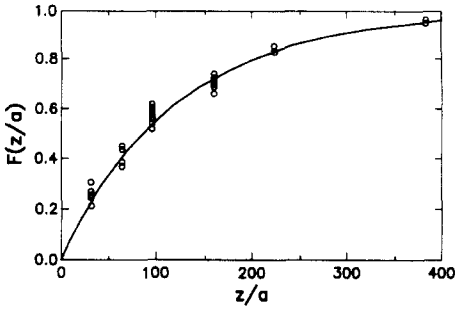

Figure 7. Comparison of the fraction deposited measured by Cousins & Hewitt (1968) with the prediction of the present model; $D_t = 0.953$ cm, Re = 56,500, $\beta\tau_{Lf} = 0.070$.

From [27], [28] and [4] it follows that

$$\frac{1}{k_D} = \frac{1}{k_1} + \frac{1}{k_2}. \qquad [29]$$

The concentration profile demonstrates that for fully developed annular flow, the concentration at the wall is just slightly larger than the bulk concentration. Therefore, the deposition process is controlled by free flight to the wall. Thus, $k_D \approx k_2$ and, according to [3], k_D scales with $(\overline{v_p^2})^{1/2}$. From [17] and [18] the deposition constant will scale with the friction velocity of the gas, as suggested by McCoy & Hanratty (1975). However, this dimensionless quantity, k_D/u_*, is a function of the droplet inertial parameter, $\beta\tau_{Lf}$. This interpretation of the mass transfer process in annular flow has not been considered previously.

Many researchers have reported measurements of k_D in annular flow. The most recent results are reported by Schadel *et al.* (1990). Deposition rates were reported, for fully developed flow, for a large range of flow conditions in three different pipe sizes. Figure 6 compares the prediction of the present model to the data of Schadel *et al.* (1990) for conditions where droplet loading is assumed dilute. Values of $\beta\tau_{Lf}$ were calculated for each of the runs of Schadel *et al.* (1990) by [2] and [16], with the drop size correlation of Azzopardi (1985) and the friction factor correlation of Asali *et al.* (1985). Good agreement is noted.

Cousins & Hewitt (1968) investigated the deposition process in annular flow by measuring deposition downstream of a section in which the film was completely removed from the wall. The flow rate of the film was measured to determine the flow rate of droplets in the gas core. The film was reformed by depositing droplets and removed again at various lengths downstream of the first removal section. In this way the fraction of liquid deposited was measured as a function of distance downstream.

Assuming fully developed flow at the point where the droplet source is turned off, the fraction of liquid deposited downstream of the shut-off is given by

$$F(t) = \frac{\int_0^t dt \int_{-\infty}^{t} -\epsilon_p(t-t') \dfrac{\partial C(r, t-t')}{\partial r} dt'}{\int_0^{\infty} dt \int_{-\infty}^{t} -\epsilon_p(t-t') \dfrac{\partial C(r, t-t')}{\partial r} dt'}. \qquad [30]$$

Here, as before, time can be converted to axial distance for the no slip assumption.

Figure 7 compares predictions with data obtained in a 0.953 cm pipe. Figure 8 makes the same comparison for data taken in a 3.18 cm pipe. Very good agreement is noted for all three experiments.

One of the interesting findings for the experiments of Cousins & Hewitt (1968) was that the deposition constant decreases sharply immediately downstream of the film removal point and approaches an asymptotic value far downstream. In terms of the present model, this effect can be attributed to a change in the concentration profile to a more diffuse shape. When the droplet source is turned off from a fully developed condition, the peak near the wall dies away and the bulk concentration becomes larger than the concentration near the wall. Because of this, both diffusive and free flight mechanisms are contributing to the resistance to deposition. Figure 9 compares the

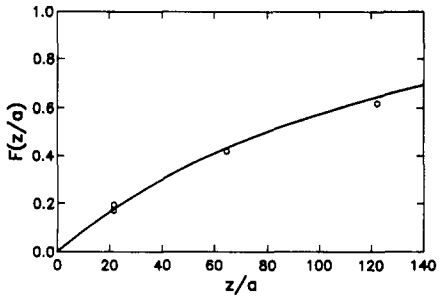

Figure 8. Comparison of the fraction deposited measured by Cousins & Hewitt (1968) with the prediction of the present model; $D_t = 3.18$ cm, Re = 200,000, $\beta\tau_{Lf} = 0.12$.

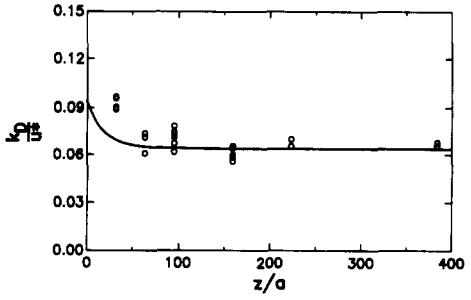

Figure 9. Comparison of the deposition constants measured by Cousins & Hewitt (1968) downstream of a film removal section with the prediction of the present model; $D_t = 0.953$ cm, Re = 56,500, $\beta\tau_{Lf} = 0.070$.

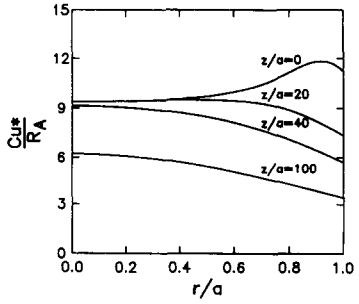

Figure 10. Concentration profiles at various locations downstream of a film removal section; $D_t = 0.953$ cm, Re = 56,500, $\beta\tau_{Lf} = 0.070$.

prediction of k_D/u_* to the data for the 0.953 cm pipe and figure 10 gives calculated concentration profiles.

7. DISCUSSION

A turbulent diffusion model has been presented for droplet deposition in vertical gas/liquid annular flow. Important ingredients of this analysis are the representation of the droplet dispersion in terms of a distribution of ring sources at the wall, the formulation of a boundary condition for the droplets at the wall and the consideration of the time dependency of the diffusion process. The turbulence characteristics of the droplets are represented in terms of the mean square of the turbulent velocity fluctuations, $\overline{v_p^2}$, a long time turbulent diffusivity, $\epsilon_{p\infty}$, and a turbulent length scale, $L = \epsilon_{p\infty}/(\overline{v_p^2})^{1/2}$. A turbulent diffusion model can be expected to work provided l is larger than the viscous wall region and smaller than the pipe radius. From the correlation of McCoy & Hanratty (1975), the first of these conditions is satisfied if $\tau^+ > 20$. From the discussion at the end of section 3 it is found that the second condition is satisfied if $\beta\tau_{Lf} > 0.01$.

The model is implemented for conditions where the relative velocity is small. Deposition is interpreted as being controlled by two resistances, diffusion to the vicinity of the wall and a free flight to the wall. Good agreement with experimental data is obtained and a number of interesting results emerge, which provide an improved understanding of annular flow.

For a fully developed condition a flat concentration profile, with a slight maximum near the wall, is obtained. Droplet deposition is controlled by free flight to the wall and the deposition constant scales with $(\overline{v_p^2})^{1/2}$. The maximum close to the wall reflects the influence of the boundary condition.

For experiments in which the film is withdrawn from the wall and deposition is measured downstream of the film removal unit, the concentration profiles change to a diffuse shape, so that the diffusional resistance becomes more important. This could explain the decrease of k_D observed by Cousins & Hewitt (1968).

In order to simplify the analysis, the velocity profile was assumed to be flat and the turbulence was assumed to be homogeneous. Of more concern, however, is the assumption that the droplets were fully entrained in the turbulence when they entered the field. Other assumptions could have

easily been made but insufficient experimental results about the atomization process were available to justify such a pursuit.

There has been considerable discussion in the literature regarding whether depositing droplets are moving in straight or zig-zag paths. It is interesting to note that the question is resolved in a natural way by considering turbulent diffusion as time dependent. Equation [6] predicts that for small diffusion times that the dispersion is approximated by $\overline{X_p^2} = \overline{v_p^2} t^2$ and that the particles are moving in straight lines. For large time it is predicted tha $\overline{X_p^2} = 2t\overline{v_p^2}\tau_p$ and that the particles move in a haphazard manner. For particles with very large inertia (small $\beta\tau_{Lf}$), the particle length scale can become comparable to the pipe diameter. In this case the droplets deposit on the wall before they deviate appreciably from the $\overline{X_p^2} = \overline{v_p^2} t^2$ behavior that they assume when they enter the gas flow.

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